Lesson Plan
Lecture Edition

Exponential Functions

Objectives

Students will:

- Graph exponential equations of the form $y = Ma^{kx}$.
- Understand the effects of '**M**', '**a**', and '**k**' on the graph
- Solve application problems using exponential functions.

Prerequisite Knowledge

Students are able to:

- Graph linear and quadratic functions.
- Solve equations for a given variable.

Resources

- This lesson assumes that your classroom has only one computer, from which you can lecture. If your classroom has enough computers for all your students, either working individually or in small groups, see the lab version of this lesson.
- Rulers, pencil, and paper, checkerboards or attachment, one bag M&Ms per student (or a big bowl full of M&Ms for the class), graph
- Access to http://www.explorelearning.com/
- Copies of the worksheet for each student (optional)

Lesson Preparation

Before conducting this lesson, be sure to read through it thoroughly, and familiarize yourself with the Exponential Functions activity at ExploreLearning.com. You may want to bookmark the activity page for your students. If you like, make copies of the worksheet for each student.

Lesson

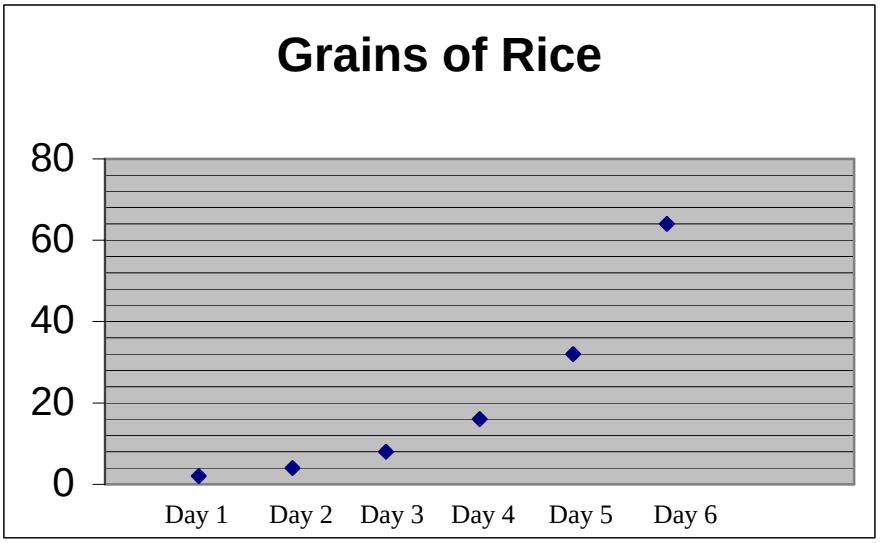
Motivation:

Warm-up

An ancient Asian story has it that a peasant worker was called before a sultan and asked to perform a difficult task. The sultan asked the peasant what would be the charge for such a service. The peasant replied that the sultan would place two grains of rice on a checkerboard for the first day's payment. On the second day he would place four grains of rice on the second square of the checkerboard. On the third day he would place eight grains of rice on the third square of the checkerboard. When the checkerboard had no more squares to fill, the peasant would take the rice as payment and nothing more.

Modeling the story

Provide each student with a checkerboard, a bag of M&Ms, and a graph. Have students place two M&Ms on the first square of a checkerboard and mark the appropriate point on their graph. Now have students place four M&Ms on the second square of the checkerboard and graph the data. When all M&Ms have been used continue graphing 'theoretical' data points through at least day six. The graph should look similar to the one below.



Have students connect the data points to obtain a graph. Ask students to describe the shape of the graph. Ask students if the graph could be modeled by a linear equation or a quadratic equation. Have students attempt to come up with a quadratic equation that models the data. Use a graphing calculator if possible to save time.

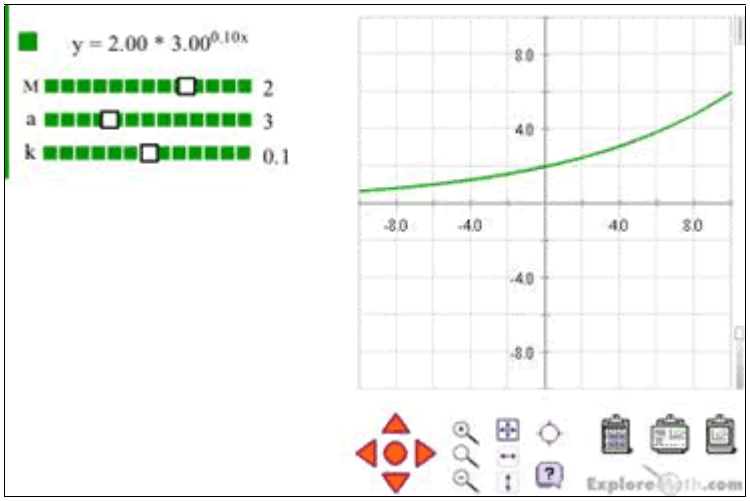
Ask students if they were successful in finding an equation to model the data. Ask them what happened to the model in the "later days." Students should see that the data increases faster than quadratic functions. This should lead nicely into a discussion of a new class of functions called exponents. Tell students they

will compute the peasant's pay later in the lesson. Now it is time to investigate exponential functions.

The exponential functions activity
Go to the Exponential Function activity at ExploreLearning.com.

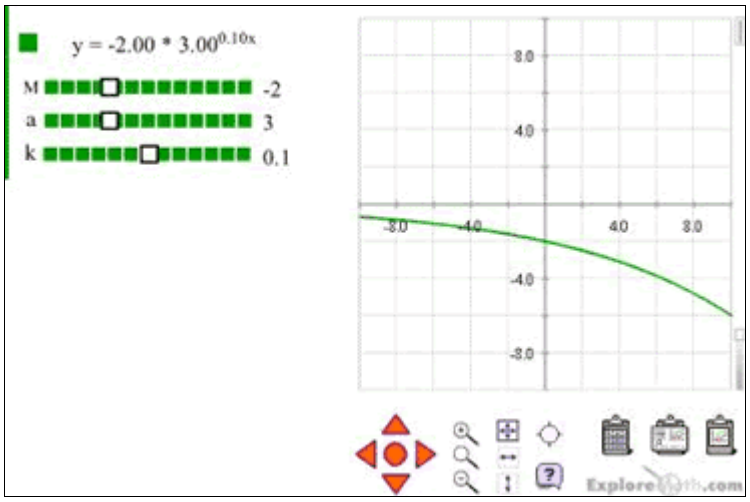
Initial values
To the right of the '**k**' slide bar, type in a **0**. Slide the '**a**' bar. Ask students what happens to the graph. Now slide the '**M**' bar. Ask students to notice the y-intercept as the value of '**M**' changes. Have students make conjectures as to why this happens. Students should see that when a base is raised to the 0 power it equals one, so we have $y = M(1) = M$.

Ask students what the value of $y = 2(3^{0.1x})$ is when $x=0$. Type in the appropriate values for '**a**', '**M**', and '**k**'. The graph will appear on the screen.



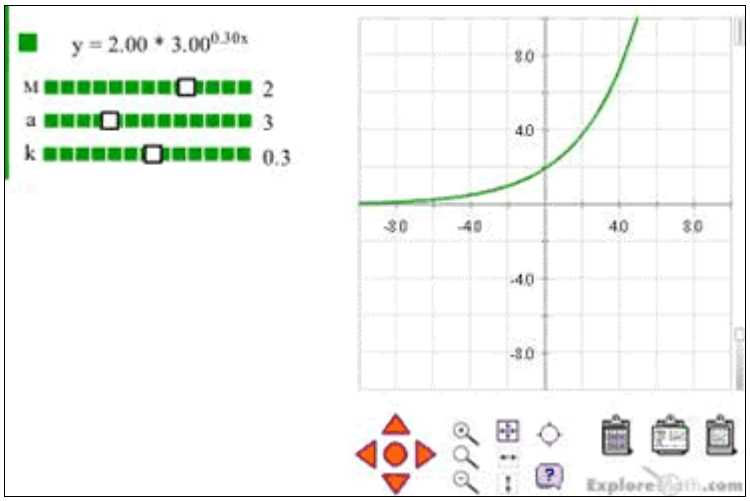
Students should see that $y=2$ when $x=0$.

Reflections in the x-axis
Slide the '**M**' bar from positive to negative. Ask Students what happens to the graph when '**M**' changes sign. Change the previous equation, $y = 2(3^{0.1x})$, to the equation $y = -2(3^{0.1x})$.



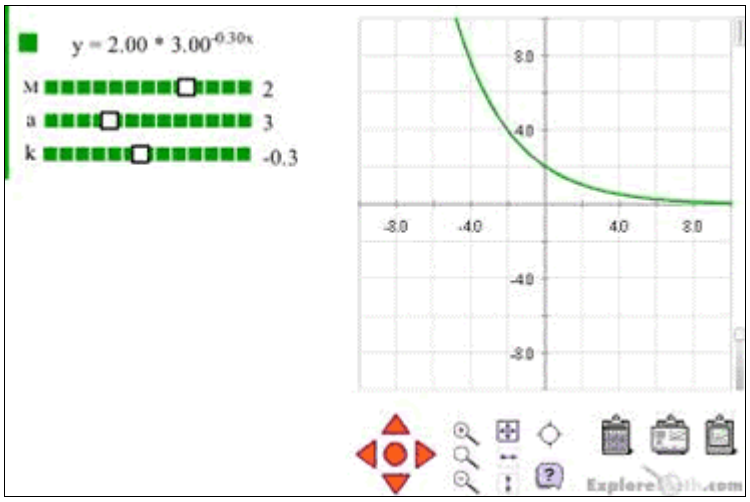
Ask students how this graph differs from the previous graph. Lead students towards the idea that the new graph is a reflection of the previous graph in the x-axis. Have students make conjectures about the sign of '**M**' and reflections.

Reflections in the y-axis and the effects of 'k'
Graph $y = 2(3^{0.3x})$.



Now slide the '**k**' bar left and right. Ask students what happens to the graph when '**k**' moves away form zero. Ask students what happens to the graph when '**k**' gets close to zero. Students should see that as 'k' gets larger, the graph gets "steeper."

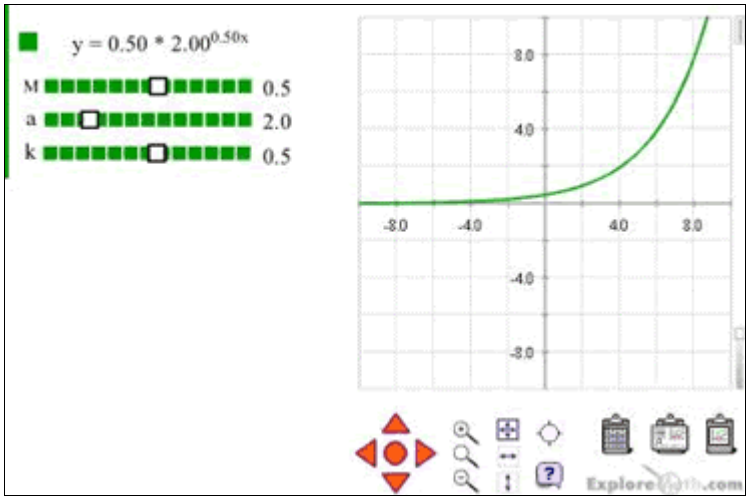
Change the equation to $y = 2(3^{-0.3x})$.



Ask students how the graph of $y = 2(3^{-0.3x})$ differs from the graph of $y = 2(3^{0.3x})$. Students should see that the graph of $y = 2(3^{-0.3x})$ is the reflection of $y = 2(3^{0.3x})$ about the y-axis. Experiment with reflections of different equations. Ask students to make conjectures about exponential equations and reflecting in the y-axis.

The effects of 'a' on the graph

Graph $y = 0.5(2^{0.5x})$



Now drag the '**a**' slide bar until **a=0**. Ask students why the graph is a horizontal line. Students should see that the equation is simplified to $y = M$.

Drag '**a**' to the left. Ask students if the graph is increasing or decreasing when '**a**' is negative. Drag '**a**' to the right. Ask students if the graph is increasing or decreasing when 'a' is positive. Ask students if the graph gets "steeper" or "flatter" as '**a**' moves away from zero. Have students make conjectures about the effects of '**a**' on the graph.

Law of uninhibited growth
Explain the following to the students in your own words.

Many natural phenomena can be modeled by the equation:

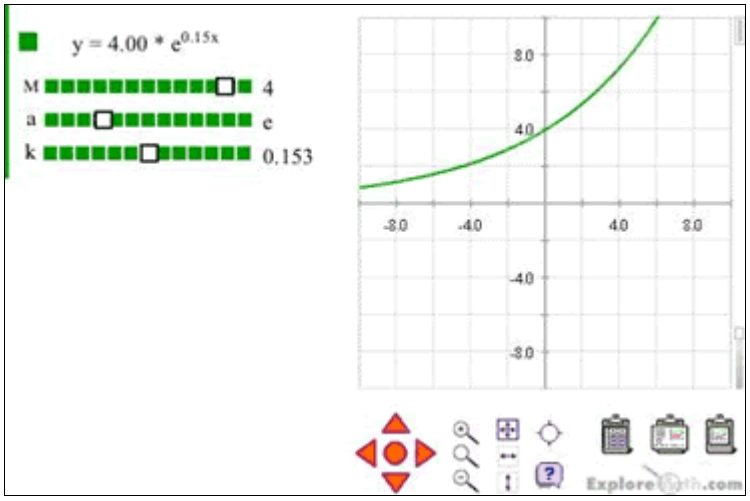
$\mathbf{A = A_0e^{kt}}$ where $\mathbf{A_0}$ is the original amount **(t=0)** and **A** is the amount after **t** years.

Let's say it takes **6** hours for a population of **4** cells to increase to **10** cells. Using this information, the following equation can be derived.

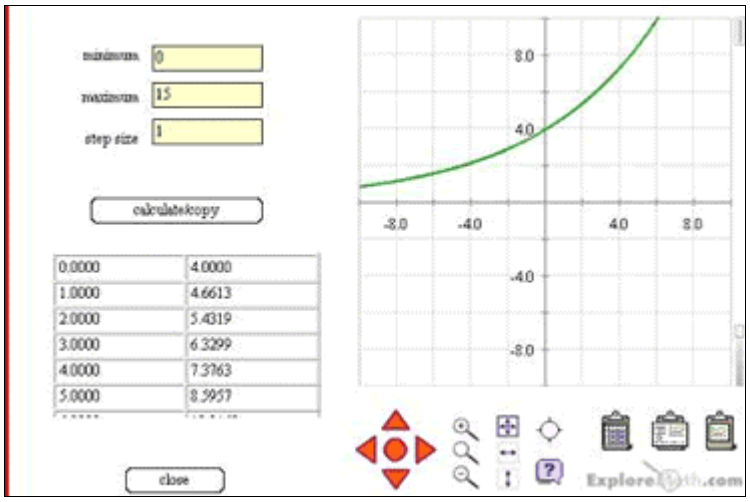
At t = 0	After **6** hours
$\mathbf{A = A_0e^{kx}}$	$\mathbf{10 = 4e^{k6}}$
$\mathbf{A = 4e^{k(0)}}$	$\mathbf{0.153 = k}$

Therefore, equation of this population is $\mathbf{A = 4e^{(0.153)t}}$.

Graph this function using the activity where **t** = **x**, **A** = **y**, and $\mathbf{A_0}$ = **M**.



Select the "Calculate Data Values" clipboard at the bottom of the screen.



Scroll through the data values to show how the cell population increases with the hours. Ask students how many cells would be present after **15** hours.

Ask students how the sign of '**k**' determines if a population model is increasing or decreasing.

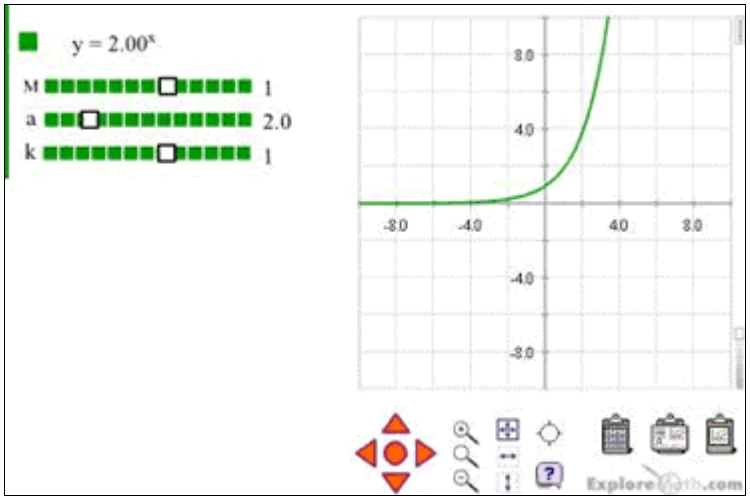
Give the students several growth/decay applications to solve.

The Rice Problem

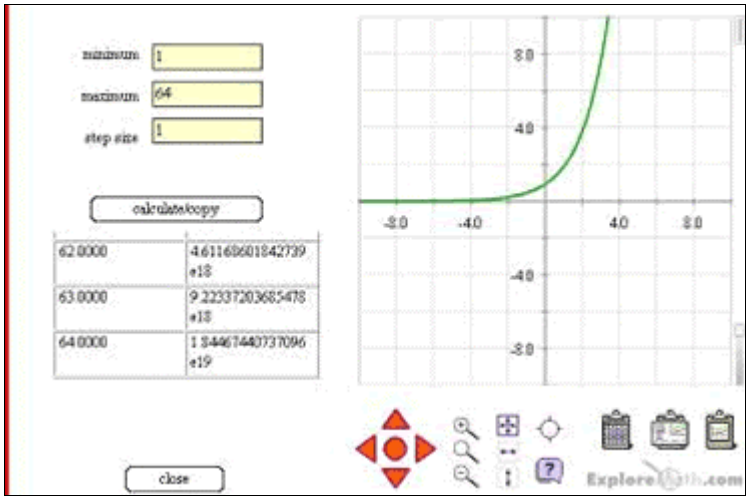
Ask students how the amount of rice is increasing each day in the problem. They should see that the rice is doubling each day. Lead students towards the idea that the amounts of rice each day are powers of **2**.

Lead students towards the equation $y = 2^x$ where y is the amount of rice put on the checkerboard on day **x**.

Graph this equation using the activity.



Now select the “Calculate Data Values” clipboard. Set the ranges from **1** to **64** with a step of 1.



Scroll through the data values to show how many grains of rice were paid each day. On just the **64th** day, the peasant should have been paid **18,446,744,073,709,600,000** grains of rice. Have students convert this number to bags of M&Ms. A **1.69**-ounce bag of M&Ms contains around **55** candies.

Conclusion

Exponential functions of the form $y = Ma^{kx}$ can be used to model certain population growths as well as data that increases or decreases rapidly. The value of ‘**k**’ can be used to reflect the function in the y-axis. The value of ‘**M**’ can be used to reflect the function in the x-axis.