

Expected Outcome (Probability)

Objectives:

Students will:

- Determine expected values of experiments
- Explore distributions of experiments

Prerequisite Knowledge

Students are able to:

- Find the area of rectangles and circles
- Determine elementary probabilities

Resources

- This lesson assumes that your classroom has enough computers for all your students either working individually or in small groups. For classrooms with only one computer, from which you can lecture, see the [lecture edition of this lesson](#).
- Rulers, pencils, paper, [histogram chart](#), and similar quarters
- Access to <http://www.explorelearning.com/>
- Copies of the [worksheet](#) for each students (optional)

Lesson Preparation

Before conducting this lesson, be sure to read through it thoroughly, and familiarize yourself with the [“Area Probability \(throw darts\)” activity](#) on [ExploreLearning.com](#). You may want to bookmark the activity page for your students. If you like, make copies of the [worksheet](#) for each student.

Lesson

Motivation

Divide the class into groups of two and give each group a quarter. Tell the groups that they are going to flip the quarter fifty times, keeping track of the

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results. Before conducting the experiment, ask the students to make a guess as to how many times the fifty flips will result in 'heads'.

Have the groups conduct the experiment, then graph all the results on a [histogram](#). Ask them if the results matched their guesses. Ask them to describe the basis on which they made their guesses. Ask them to describe the characteristics of the histogram.

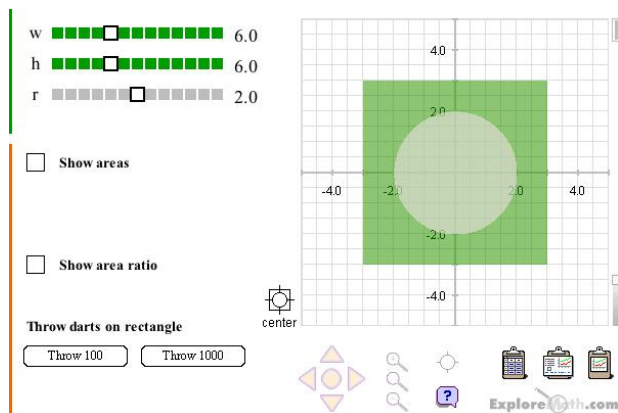
A lot of the guesses were probably around 25 heads. Explain to the students that 25 was the 'expected outcome' for the number of heads. Tell the students that they will explore expected outcomes and probability distributions in the following activity.

The Area Probability (throw darts) activity

Have the students go to the ["Area Probability \(throw darts\)" activity](#) on [ExploreLearning.com](#).

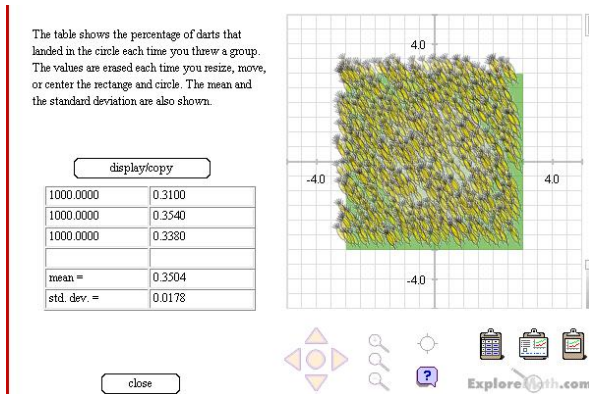
Theoretical versus Experimental probability

When the activity loads up, it will look like the picture below.



Have the students run an 'experiment' by throwing 1000 darts ten different times. Once this task is completed have them select the 'calculated data values' clipboard and find the mean percentages of darts that landed in the circular region.

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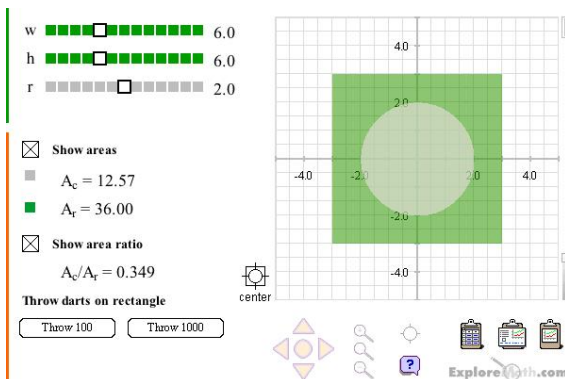


In the example above, 35.04% of the darts landed in the circular region.

Ask the students what percent chance a dart has of landing in the circular region according to this experiment. Lead them towards the idea that a dart has a 35.04% chance of landing in the circular region.

Explain to the students that this is an example of experimental probability. Experimental probability is probability based on previous trails or experiments.

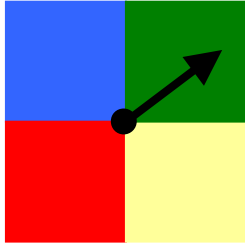
Now have the students select the 'Show area' and the 'Show area ratio' boxes.



Ask the students to describe the significance of the area ratio. Lead students towards the idea that this ratio is the theoretical probability that a dart will land in the circular region. In this case a dart theoretically has a 34.9% chance of landing in the circular region.

Ask the students which type of probability would be used to determine the probability that a baseball player will get a hit her next time at bat.

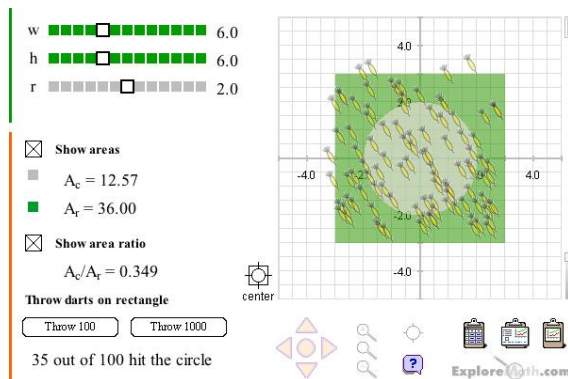
Ask the students what type of probability they would use to determine the probability that the spinner pictured below would land in the blue section after one spin.



The class may have been divided as to what type of probability to use for the spinner. Ask the students to compare the strengths and weaknesses of each type of probability. Lead the students towards the idea that after repeated trials, the experimental probability should approach the value of the theoretical probability.

Expected outcome

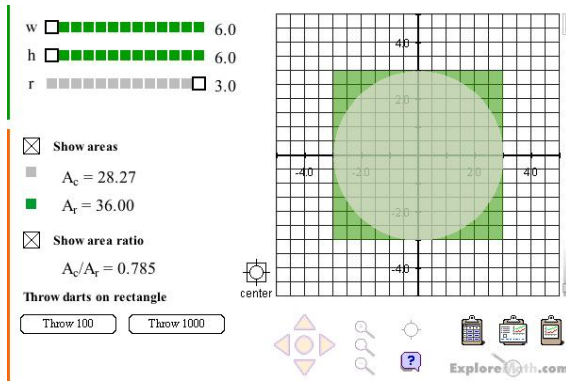
Ask the students to predict how many darts out of 100 will land in the circular region, then select the 'Throw 100' radio button.



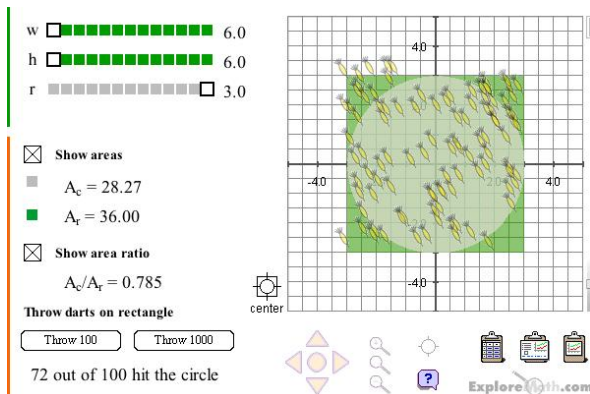
Go over the results. In the case above 35 darts landed in the circular region.

Tell the students that expected outcome, $E(x)$, is the number of 'successes' or the 'value' of an experiment. Ask the students how they think expected outcomes are calculated. Lead students towards the idea that $E(x) = (\text{probability of success with one trial}) \times (\text{number of trials})$.

Have the students set the radius of the circle to 3.

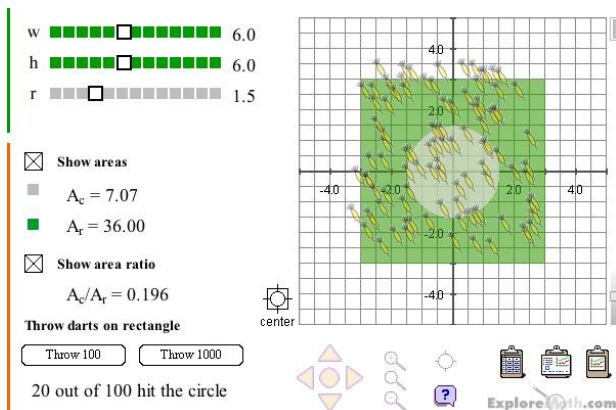


Now have the students calculate the expected outcome when 100 darts are thrown. In this case $E(x) = 78.5$, so it is expected that around 78 to 79 darts will land in the circular region. Have them conduct several trials of throwing 100 darts to see if the outcome is around 78 or 79 darts. In the picture below the outcome was 72 darts.



Challenge

Have the students try to get 20 darts in the circular region. Adjusting the sizes of each region is the key to accomplishing this. Go over the results of this challenge. Below is a picture of a successful throw. Notice the ratio of the areas.



Probability distributions

Normally distributed populations have certain characteristics. We will explore these characteristics using histograms and this activity.

Have the students set the radius of the circle to 3, then conduct 50 trials of 100 dart throws. After completing the throws select the 'calculate data values' clipboard at the bottom of the activity.

The table shows the percentage of darts that landed in the circle each time you threw a group. The values are erased each time you resize, move, or center the rectangle and circle. The mean and the standard deviation are also shown.

display/copy

throws	% in circle
100.0000	0.7500
100.0000	0.8000
100.0000	0.8100
100.0000	0.8300
100.0000	0.7700

close

The results of the thirty trials are contained in this table. Use the table to calculate the number of darts landing in the circular region for each throw. Have the students plot the results on a histogram.

Ask the students the following questions about their histogram:

What is the range of data points?

Why are no data points located outside this range?

What data points had the highest frequency?

Were these points located near the expected outcome of the experiment? Why?

Repeat this exercise for a different sized circle. Also have the students look at the histogram made at the beginning of the lesson with the coins.

Have the students make conjectures about the characteristics of normal distributions. Lead them towards the idea that the frequency of data points in normal distributions is greater near the expected outcome of the experiment. The frequency declines when the data point is farther away from the expected outcome. In a true normal distribution the frequency histogram is in the shape of a bell.

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Conclusion

Expected outcomes of trials can be determined by multiplying the probability of a success of one trial by the number of trials conducted. When several trials are performed, the frequency of the outcomes should be greater near the expected outcome.