Teacher Guide: Modeling Decimals (Base-10 Blocks)

Learning Objectives

Students will…

- Model whole numbers using base-10 blocks.
- Model decimals using base-10 blocks.
- Model a single number in several different ways.
    - For example, 32 can be represented with 3 rods and 2 cubes, or with 2 rods and 12 cubes.
- Model regrouping with base-10 blocks.

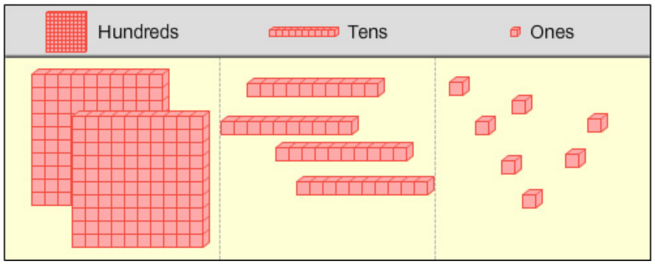
Vocabulary

base-10 blocks, base-10 system, decimal, decimal point, whole number

Lesson Overview

The *Modeling Decimals (Base-10 Blocks)* Gizmo™ allows students to model numbers using base-10 blocks. Unit blocks can be used individually or grouped into rods of 10 or flats of 100.

Students can model whole numbers or decimals using the same set of blocks. Regrouping can be modeled by exchanging unit cubes for rods, rods for flats, and so on.



Modeling 247 with base-10 blocks

The Student Exploration sheet contains three activities:

- Activity A – Students model whole numbers between 1 and 999.
- Activity B – Students model decimals between 0.01 and 9.99.
- Activity C – Students use the blocks to model decimals from 0.001 to 0.999.

Suggested Lesson Sequence

1. **Pre-Gizmo activity** (10 – 20 minutes)
   If possible, obtain a classroom set of base-10 blocks. (You could also cut out flats, rods, and squares from graph paper.) Practice creating different numbers from 1 to 999 using the blocks. Stress the relationship between each type of block and the digits that they represent. For example, 216 is represented by 2 flats (200), 1 rod (10) and 6 cubes (6).

2. **Prior to using the Gizmo** (10 – 15 minutes)
   Before students are at the computers, pass out the Student Exploration sheets and ask students to complete the Prior Knowledge Questions. Discuss student answers as a class. At this point, letting students share how they thought about the questions is more valuable than "going over" the correct answers. After the discussion, if possible, use a projector to introduce the Gizmo and demonstrate its basic operations.

3. **Gizmo activity** (15 – 20 minutes per activity)
Assign students to computers. Students can work individually or in small groups. Have students work part of the Student Exploration sheet using the Gizmo. Alternatively, you can use a projector and do the Exploration as a teacher-led activity.

It may be overwhelming for students to do all of the activities in the Student Exploration in one sitting. We recommend starting with the first page of the Student Exploration sheet (Prior Knowledge Questions and Gizmo Warm-up) plus one of the two activities. Extend the lesson if you want using the extensions below. Return to the Gizmo and the unused activities in future class periods to reinforce the concepts.

4. **Extending the Gizmo** (10 – 15 minutes each)
Here are some suggestions for extending the activities in the Student Exploration sheet.

Activity A Extension – Continue to practice modeling numbers with the Gizmo. Ask students how many different ways they could use the Gizmo to model a number like 313. (There are 68 possible ways to model 313 using the Gizmo!)

Activity B Extension – A common misconception with decimals is to ignore the decimal point when comparing decimals. For example, some students may think that 0.27 is greater than 0.9 because 27 is greater than 9. You can use the Gizmo to dispel this misconception because 9 rods are clearly more than 27 cubes. (You can also convert the 9 rods into 90 cubes.)

Activity C Extension – After learning to model numbers with the *Modeling Decimals* Gizmo, the next step is to use base-10 blocks to add and subtract whole numbers and decimals. You can do this with base-10 blocks in the classroom, or use the *Adding Decimals* and *Subtracting Decimals* Gizmos. (Note: *Subtracting Decimals* will be published by January 2009.)

5. **Follow-up activity: Binary numbers** (30 – 60 minutes)
The base-10 system is the number system used by people all over the world. Students may be surprised that the base-10 system is not the only way to write numbers. Computers use the base-2, or **binary**, system. (This is because each tiny circuit in a computer has just 2 states: "on" or "off.")

There are two major differences between binary numbers and base-10 numbers. Binary numbers only use two digits, 0 and 1. (The base-10 system uses 10 digits, 0 – 9.) Each place in a binary number represents a power of 2, rather than a power of 10. The right digit of a binary number is the 2^0 (1's) place, the next digit is the 2^1 (2's) place, the next digit is the 2^2 (4's) place, etc. Here is the number 19 written in base-10 and binary form:

Base-10

100	10	1
0	1	9

Binary

16	8	4	2	1
1	0	0	1	1

So the binary number 10011 means 16 + 0 + 0 + 2 + 1 = 19. The binary number 1010 is 8 + 0 + 2 + 0 = 10. Have your students practice writing binary numbers. If you like, try adding and subtracting these numbers as well.

Mathematical Background

Because we have all grown up with the base-10 system, it seems a completely natural way to write numbers. But there is nothing special about the number 10. If we had a different number of fingers on our hands, it is likely that people would have adopted a different system for counting numbers. Throughout history, many cultures have used other base systems for counting:

- Primitive tribes such as the Piraha of South America lack specific words for numbers higher than 2 or 3.
- Some cultures, including the Yuki tribe of California, have used base-8 systems.
- The Maya used a base-20 system, probably based on the number of fingers and toes.
- The ancient Babylonians used a base-60, or *sexagesimal* system.

Ancient Egyptians were one of the earliest cultures to use a base-10 system. They used the hieroglyphs "𓏺" for 1, "𓎆" for 10, and "𓍢" for 100. The number 342 in hieroglyphics is shown at right. Similar numerical systems were used in the Greek and Roman civilizations.

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Our modern numeral system was originally developed in India around 500 AD. Key innovations included the significance of position, the invention of zero, and the use of a decimal symbol to divide the ones place from the tenths place. This allowed numbers to be written with a minimum number of digits, and greatly simplified arithmetic. The system was adopted by Arabs, and then brought to Europe in the 10^{th} Century. Because they were introduced by Arabs, these numerals became commonly known as "Arabic numerals."

In the base-10 system, each place represents a multiple of a power of 10. For example, the number 2,509 can be broken down as shown below. (Note: Any number raised to the zero power, such as 10^0, equals 1.)

$$2{,}509 = (2 \times 10^3) + (5 \times 10^2) + (0 \times 10^1) + (9 \times 10^0) = 2{,}000 + 500 + 0 + 9$$

Base-10 blocks are useful tools for understanding the base-10 system because they are a physical representation of the numbers they represent. Each rod contains 10 cubes, and each flat contains 10 rods (100 cubes). It is relatively easy to understand that 276 is two flats (2 hundreds), seven rods (7 tens), and 6 cubes (6 ones). Similarly, 5.13 can be represented by 5 flats (5 wholes), 1 rod (one tenth), and 3 cubes (3 hundredths).

Base-10 blocks can also be used to show "carrying" during addition and "borrowing" during subtraction. For example, if there are more than 10 cubes, the 10 cubes can be exchanged for a rod of 10 cubes. Similarly, a rod can be broken up into 10 cubes to show borrowing.

Selected Web Resources

Using base-10 blocks: http://curriculalessons.suite101.com/article.cfm/teaching_place_value
Teaching place value: http://www.arcytech.org/java/b10blocks/counting.html
Base-10 blocks simulation: http://www.learningbox.com/Base10/BaseTen.html
Bear catches blocks: http://www.learningbox.com/base10/CatchTen.html
Binary numbers: http://www.math.grin.edu/~rebelsky/Courses/152/97F/Readings/student-binary
Binary number game (challenging!): http://forums.cisco.com/CertCom/game/binary_game.swf
Numeral systems: http://en.wikipedia.org/wiki/Numeral_system
Adding Decimals Gizmo: http://www.explorelearning.com/gizmo/id?1023