Teacher Guide: Function Machines 3

Learning Objectives

Students will…

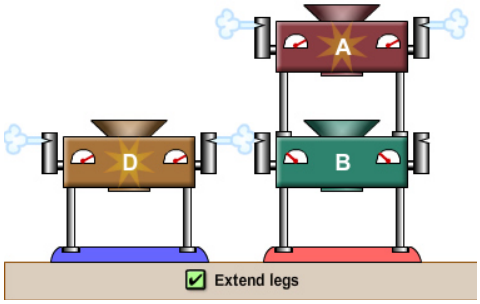
- Model and solve word problems using function machines.
- Combine two functions to model a real-world situation.
- Predict the output of a combination of machines.
- Use function machines to demonstrate inverse operations:
  - Subtraction is the inverse of addition (and vice versa).
  - Division is the inverse of multiplication (and vice versa).

Vocabulary

function, input, inverse operation, output

Lesson Overview

The *Function Machines 3* Gizmo™ is the third in a sequence of Gizmos that introduce students to functions. (For links to *Function Machines 1* and *2*, see **Selected Web Resources** on page 3 of this document.)



The *Function Machines 3* lesson focuses on using functions to model real-world situations such as taxi fares or phone plans. Students can solve a variety of word problems by combining two or more machines.

The Student Exploration sheet contains three activities:

- Activity A – Students model taxi fares using function machines.
- Activity B – Students explore inverse operations.
- Activity C – Students use function machines to solve word problems.

Suggested Lesson Sequence

1. **Pre-Gizmo activity** (10 – 15 minutes)

   The *Function Machines 3* Gizmo was designed as a follow-up to the *Function Machines 1* and *Function Machines 2* Gizmos. We recommend doing those activities before attempting this one.

   To introduce the concepts discussed in the *Function Machines 3* Gizmo, have students try to solve word problems such as the two examples below. For each problem, have students explain how they found the solution. Then ask them to describe a function that will allow them to solve the problem no matter what the input number is.

   - A monkey eats 5 bananas every day. How many bananas does it eat in 11 days?
   - Janice was 6 when her little brother Henry was born. How old will Henry be when Janice is 21?

2. **Prior to using the Gizmo** (10 – 15 minutes)
Before students are at the computers, pass out the Student Exploration sheets and ask students to complete the Prior Knowledge Questions. Discuss student answers as a class. At this point, letting students share how they thought about the questions is more valuable than "going over" the correct answers. After the discussion, if possible, use a projector to introduce the Gizmo and demonstrate its basic operations.

3. **Gizmo activity** (15 – 20 minutes per activity)
Assign students to computers. Students can work individually or in small groups. Have students work part of the Student Exploration sheet using the Gizmo. Alternatively, you can use a projector and do the Exploration as a teacher-led activity.

It may be overwhelming for students to do all of the activities in the Student Exploration in one sitting. We recommend starting with the first page of the Student Exploration sheet (Prior Knowledge Questions and Gizmo Warm-up) plus one of the three activities. Extend the lesson if you want using the extensions below. Return to the Gizmo and the unused activities in future class periods to reinforce the concepts.

4. **Extending the Gizmo** (15 – 20 minutes)
Here are some suggestions for extending the activities in the Student Exploration sheet:

Activity A Extension – Ask some follow-up questions about the three cab companies, Acme, Beta, and Champion. For what range of distances is each company the best deal? (Acme is cheapest for distances of 0 – 5 miles, and Beta is cheapest for distances of 6 or more miles. Champion ties Acme for a 5 mile trip, and ties Beta for a 6 mile trip.)

Activity B Extension – In each of the problems in this activity, the order of the machines is critical. For example, if a phone plan charges $20 per month plus $2 per minute, it is essential to multiply the minutes by 2 first, and then add 20. Explore how order matters in each problem by having students stack the machines in reverse order.

Ask, when does the order of machines matter, and when does it not matter? (Order only matters when an addition or subtraction machine is combined with a multiplication or division machine. It does not matter when both machines do addition or subtraction, or when both machines do multiplication or division.) For more on order of operations, try the *Order of Operations* Gizmo. (See **Selected Web Resources** on page 3 for the link.)

Activity C Extension – Challenge students to play a function machines guessing game. Have students work in pairs. One person programs two machines, and the other tries to figure out the function rule by making an input-output table. Ask students about their strategies. What helps them figure out the functions most quickly? With two machines, what is the minimum number of input-output pairs they need to determine the function?

5. **Follow-up activity: Invent your own problem** (30 – 60 minutes)
Group students in pairs. Each pair has to come up with a real-world problem that can be solved using the function machines in the Gizmo. To present their problem, have each group create a small poster. The poster should contain a title, the problem, and the function machines that can be used to solve it. Students can also draw pictures on the poster or print pictures from the internet to illustrate the problem. After all the pictures are displayed, students can vote on their favorites.

Mathematical Background

Solving word problems is a very important skill for a student to learn. The *Function Machines 3* activity was designed to help students build problem-solving strategies by using mathematical models.

To model a real-world situation, pay attention to the language used to describe the situation. Words like “increased by,” “total,” and “more” may indicate addition, while words like “less than,” and “difference” may indicate subtraction. Words like “per” and “out of” may indicate division, and words like “of” and “times” may indicate multiplication.

Consider the case of the Champion Taxi Service described in the Gizmo. Champion cabs charge a set fee of $5 when you get in the cab, plus $2 per mile. In this function, the input is the number of miles you travel, and the output is the price for the trip. Two function machines can be combined to model this situation: × 2 for the cost per mile, and + 5 for the set fee. If you travel 10 miles, your cost is 10 × 2, or $20, plus the $5 fee. Thus the total is $25.

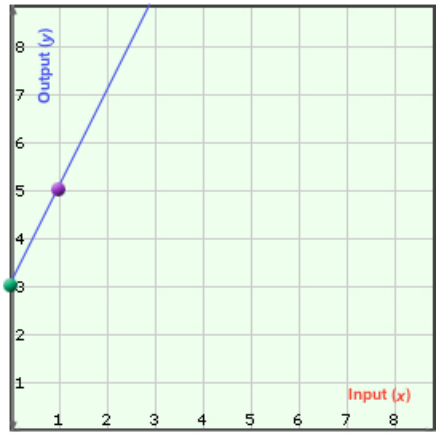
As you stack the machines in the Gizmo, you have to be very careful about the order. If you put the + 5 machine on top, you will get a very different answer (in this case, (10 + 5) × 2 = $30). Instead, what you need to do is multiply the input by 2 first, and then add 5 to the product. The only way to avoid this potential confusion is to pay attention to how the problem is phrased!

If you have a combination of machines, it is possible to use an input-output table to determine the overall function rule. In this Gizmo, all functions that you can create are *linear*, meaning they form a line when you graph the input-output pairs.

A line is uniquely defined by two points. Because of this, you can figure out any linear function just by knowing two points that are on the line (two input-output pairs).

Any linear function can be written as: output = m × input + b. The variable b is the output when the input is 0. On a graph, b is the y-intercept (the place where the line crosses the y-axis). The variable m is the *slope* of the line. The slope is how much the output changes when the input increases by 1.

The line at right has a y-intercept of 3 and a slope of 2, so it graphs the function: output = 2 × input + 3. In the Gizmo, this is modeled with a × 2 machine on top of a + 3 machine.



Selected Web Resources

Function Machines 1 Gizmo: http://www.explorelearning.com/gizmo/id?1035
Function Machines 2 Gizmo: http://www.explorelearning.com/gizmo/id?1039
Order of Operations Gizmo: http://www.explorelearning.com/gizmo/id?255
Functions: http://www.purplemath.com/modules/fcns.htm
Problem-solving tips: http://www.purplemath.com/modules/translat.htm

Other function machine interactive simulations:

http://nlvm.usu.edu/en/NAV/frames_asid_191_g_3_t_1.html
http://www.mathplayground.com/FunctionMachine.html
http://pbskids.org/cyberchase/games/functions/functions.html