Teacher Guide: Factor Trees

Learning Objectives

Students will…

- Understand that some numbers, called prime numbers, have only two factors – one and themselves.
- Understand that other numbers, called composite numbers, have multiple prime factors.
- Factor composite numbers into prime numbers.
- Build composite numbers by multiplying primes.
- Use factors to understand multiples and divisibility.

Vocabulary

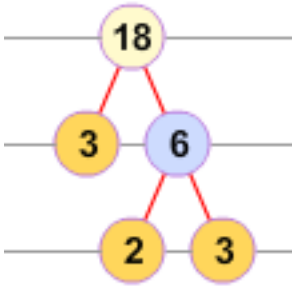
composite number, divisible, factor, factor tree, multiple, prime factorization, prime number, product

Lesson Overview

The *Factor Trees* Gizmo™ allows students to factor composite numbers into primes, using factor trees. In addition, students can "build" composite numbers by multiplying primes.

The Student Exploration sheet contains three activities:

- Activity A – Students factor numbers into prime numbers using interactive factor trees.
- Activity B – Students build numbers by multiplying primes, and explore multiples of numbers.
- Activity C – Students use factors and factor trees to learn about divisibility.



Suggested Lesson Sequence

1. **Pre-Gizmo activity** (10 – 20 minutes)
   Ask students to give you pairs of whole numbers that multiply to equal 24. There are several different pairs that students will come up with: 1 and 24, 2 and 12, 3 and 8, and 4 and 6. These are *factor pairs* for the number 24.

   Then keep going. Find factor pairs for any of those numbers. (Stay away from pairs that include a 1, as that doesn't really tell you anything interesting – any number times 1 equals itself.) For example, using the factor pair 3 and 8, the 8 can be broken down into 2 × 4. So now you have 24 = 3 × (2 × 4).

   Can you go further? (Yes, the 4 can still be broken down into 2 × 2.) What is the product when it is broken down as far as you can go? (You should get 24 = 2 × 2 × 2 × 3.)

   Then try this again, using a different factor pair of 24 as a starting point. What is the final product in that case? Help your students see that, no matter which factor pair you begin with, you should end up with the same product in the end, 24 = 2 × 2 × 2 × 3.

2. **Prior to using the Gizmo** (10 – 15 minutes)
Before students are at the computers, pass out the Student Exploration sheets and ask students to complete the Prior Knowledge Questions. Discuss student answers as a class. At this point, letting students share how they thought about the questions is more valuable than "going over" the correct answers. After the discussion, if possible, use a projector to introduce the Gizmo and demonstrate its basic operations.

3. **Gizmo activity** (15 – 20 minutes per activity)
Assign students to computers. Students can work individually or in small groups. Have students work part of the Student Exploration sheet using the Gizmo. Alternatively, you can use a projector and do the Exploration as a teacher-led activity.

It may be overwhelming for students to do all of the activities in the Student Exploration in one sitting. We recommend starting with the first page of the Student Exploration sheet (Prior Knowledge Questions and Gizmo Warm-up) and working through the activities in order. Extend the lesson if you want using the extensions below. Return to the Gizmo and the unused activities in future class periods to reinforce the concepts.

4. **Extending the Gizmo** (15 – 20 minutes each)
Here are some suggestions for extending the activities in the Student Exploration sheet:

Activity A Extension – Factor more numbers down to their prime factors. Extend this for numbers up to 100. (By using **Random** under **Number to factor** at the top left, you can get numbers up to 100. You can also type numbers in directly and click **Make**.) Stress that, no matter which factor pair you start with, you get the same prime factorization in the end. To see this, factor a number with many factors, like 48. See how many different factor pairs you can begin with. At the end, you will get the same prime factorization.

Activity B Extension – Discuss the "Multiples" sets of numbers in Activity B in the Student Exploration sheet. Be sure that students see that the same "tricks" that they find to tell if a number is a multiple of 5 (for example) will also tell them about divisibility by 5.

- All even numbers (numbers that end in 0, 2, 4, 6, and 8) are divisible by 2.
- All numbers that end in 0 or 5 are multiples of 5, and are divisible by 5.
- All numbers that end in 0 are multiples of 10 (and also of 2 and 5) and are divisible by 10 (and also by 2 and 5).
- All numbers whose digits add up to a number that is divisible by 3 are themselves divisible by 3. For example, 78 is divisible by 3 because 7 + 8 = 15, and 15 is divisible by 3. (Also, the 1 and 5 in 15 add to 6, also divisible by 3.)
- All numbers whose digits add up to a multiple of 9 are divisible by 9. For example, 342 is divisible by 9 because 3 + 4 + 2 = 9.

The **Build** mode of the Gizmo also illustrates the fact that all composite numbers can be built by multiplying primes. Can your students build all composite numbers up to 50?

Activity C Extension – Simplify some fractions that are not in lowest terms by factoring the numerator and denominator, and then by dividing the numerator and denominator by common factors. For example, use this method to show that $\frac{6}{24}$ is equal to $\frac{1}{4}$. (See the **Mathematical Background** on the next page for more information on this.)

5. **Follow-up activity** (20 – 30 minutes)
   Use the **Build** mode of the Gizmo to build some large numbers. (The Gizmo handles numbers up to 999.) Then turn this around to find the prime factorizations of these large numbers. Be sure students see that these are two ways to look at exactly the same concept. By building a number out of primes, you are determining its prime factorization.

   You can also have fun with a factoring contest. Call out a challenging number that can be factored in the Gizmo, like 630. First person to factor the number completely wins!

Mathematical Background

Factoring numbers into their prime number "building blocks" is an important skill for a number of reasons. Understanding this will improve students' number sense, get them ready for simplifying fractions, and also prepare them for factoring polynomials in algebra classes in the future.

Students should begin by getting a handle on the difference between prime numbers and composite numbers. You can do this by getting students to think about *factor pairs*.

- Factor pairs for 24: 1 and 24, 2 and 12, 3 and 8, 4 and 6
- Factor pairs for 25: 1 and 25, 5 and 5 (The number 25 is a *perfect square*.)
- Factor pairs for 29: 1 and 29

Numbers (such as 24 and 25) that have more than one factor pair are *composite numbers*. Numbers (such as 29) that have only one factor pair (1 and itself) are *prime numbers*.

As illustrated in the suggested **Pre-Gizmo activity** (on page 1 of this document), if you go far enough with factor pairs, eventually you have a product of nothing but primes. This is the *prime factorization* of the number. No matter which factor pair you start with, the prime factorization you end up with for a given number is always the same.

Prime factorizations can help students simplify fractions. Knowing what prime number "building blocks" make up numbers makes simplifying much easier. Here is one example:

$$\frac{6}{24} = \frac{2\times 3}{2\times 2\times 2\times 3} = \frac{1}{2\times 2} = \frac{1}{4}$$

Simplifying the fraction $\frac{6}{24}$ down to its fully simplified (or "lowest terms") form of $\frac{1}{4}$ requires dividing the numerator and denominator both by 6. The reason this works is that both the numerator and denominator have a "2 × 3" in them. Therefore, both can be divided by 2 × 3, or 6. (This number, 6, is called the *greatest common factor*, or GCF, of 6 and 24.)

Selected Web Resources

Finding Factors with Area Models Gizmo: http://www.explorelearning.com/gizmo/id?218
Factoring information: http://www.mathematical-factoring.info/
Factoring numbers by hand: http://www.purplemath.com/modules/factnumb.htm
How to factor a number: http://www.wikihow.com/Factor-a-Number
Factor trees lesson: http://www.tburg.k12.ny.us/anderson/ppt.htm
Factor pairs calculator: http://www.csgnetwork.com/mthfactorpairs.html