

# Teacher Guide: Balancing Blocks



## Learning Objectives

Students will...

- Find the volume of a rectangular prism by counting how many unit blocks it contains.
- Find the volume of a rectangular prism by multiplying length  $\times$  width  $\times$  height.
- Find the volume of irregular solids that are composed of unit blocks.
- Balance rectangular prisms and irregular solids on a point.



## Vocabulary

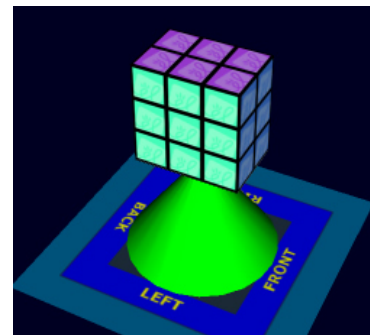
area, balance, dimension, prism, product, rectangular prism, volume



## Lesson Overview

The *Balancing Blocks* Gizmo™ gives students two challenges: to build a structure with a given number of blocks, and to build a structure that balances on the tip of a cone.

In the Gizmo, students can add or delete blocks from their structure. They can also turn the platform for a different perspective, or change the dimensions of the platform.



The Student Exploration sheet contains three activities:

- Activity A – Students find the volume of small rectangular prisms by counting blocks.
- Activity B – Students calculate the volume of larger prisms by multiplying dimensions.
- Activity C – Students try to build and balance irregular shapes.



## Suggested Lesson Sequence

### 1. Pre-Gizmo activity (🕒 15 – 30 minutes)

To prepare students for finding the volume of a rectangular prism, start by reviewing how to find the area of a rectangle. Pass out graph paper, and ask students to draw a series of rectangles. For each rectangle, have students count the squares and then multiply the dimensions. Students will see that the area of a rectangle is equal to its length  $\times$  width.

To demonstrate the concept of volume, show students a variety of cardboard boxes. Volume is a measure of how much space an object occupies. Which box occupies the most space? How would you compare the volumes of two similar boxes? Based on what you learned about rectangles, how would you guess that you find the volume of a box?

### 2. Prior to using the Gizmo (🕒 10 – 15 minutes)

*Before* students are at the computers, pass out the Student Exploration sheets and ask students to complete the Prior Knowledge Questions. Discuss student answers as a class. At this point, letting students share how they thought about the questions is more valuable than “going over” the correct answers. After the discussion, if possible, use a projector to introduce the Gizmo and demonstrate its basic operations.

3. **Gizmo activity** (🧠 15 – 20 minutes per activity)  
Assign students to computers. Students can work individually or in small groups. Have students work part of the Student Exploration sheet using the Gizmo. Alternatively, you can use a projector and do the Exploration as a teacher-led activity.

It may be overwhelming for students to do all of the activities in the Student Exploration in one sitting. We recommend starting with the first page of the Student Exploration sheet (Prior Knowledge Questions and Gizmo Warm-up) plus one of the three activities. Extend the lesson if you want using the extensions below. Return to the Gizmo and the unused activities in future class periods to reinforce the concepts.

4. **Extending the Gizmo** (🧠 15 – 20 minutes each)  
Here are some suggestions for extending the activities in the Student Exploration sheet.

Activity A Extension – Ask students to find the volume of small rectangular prisms, such as  $2 \times 6 \times 2$ , or  $3 \times 3 \times 7$ . (Note: These are both just out of the range of the Gizmo, as they each have one measurement that exceeds 5 units.) Use the “count blocks” animation in the Gizmo to motivate the logic behind finding the volume of rectangular prisms: first count how many blocks are in the first layer, and then add up the blocks in each of the layers. This will prepare students for the volume formula that they will discover in Activity B.

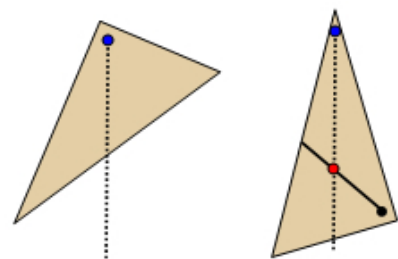
Activity B Extension – Use length  $\times$  width  $\times$  height to calculate the volumes of prisms too large to be built in the Gizmo. (You may want to let students use calculators for these.)

Activity C Extension – After finishing Activity C, students can continue solving challenges with the Gizmo for as long as they like. Students can also create their own challenges for other students to solve.

If you like, discuss with your class why it is impossible (in the Gizmo) to balance an odd number of blocks on a base with an even area. Because there is no one “center square” on the platform, no matter what you do, the last block you place will disrupt the balance.

5. **Follow-up activity: Center of mass** (🧠 30 – 45 minutes)  
The *Balancing Blocks* Gizmo has two themes: volume and balance. For a 3-dimensional object, the balance point is called the *center of mass*. To demonstrate center of mass, ask your students to cut out a variety of regular and irregular shapes from cardboard. Ask your students if they can balance the shapes on the tip of a finger or on the eraser end of a pencil. The point at which the shape balances is its center of mass.

To find the exact center of mass, tack the shape to a bulletin board so that it hangs freely. Draw a vertical line from the tack through the shape, as shown at right. Then hang the shape from another point, and draw a second vertical line. The center of mass (•) is located where the two lines cross.

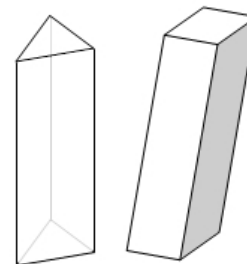


Try this out, and see if the shape can be balanced on this point. Ask your students if they can explain why this method works.



## Mathematical Background

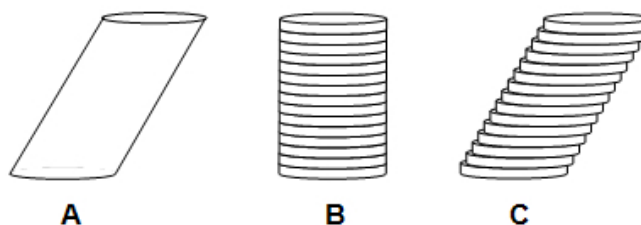
In mathematics, a *prism* is defined as a shape that has congruent bases that are connected by parallel edges. Two examples of prisms are shown at right. The *triangular prism* on the left has bases that are triangles. The *rectangular prism* on the right has bases that are rectangles. The faces of a prism can either be rectangles or parallelograms. (Non-rectangular faces happen if the prism is *skewed*, or tilted to one side or another.)



For any prism, the volume is equal to the base area multiplied by the height. In a rectangular prism the base area equals length  $\times$  width, so the volume is length  $\times$  width  $\times$  height, or  $V = lwh$ . This can be demonstrated with the rectangular prisms you can build in the *Balancing Blocks* Gizmo. The prism at left has a length of 2 units and a width of 4 units. The base area is  $2 \times 4 = 8$  units, so each layer contains 8 blocks. There are 3 layers of blocks, so there is a total of  $8 + 8 + 8 = 8 \times 3 = 24$  blocks. This is also the volume of the prism.

The formula “volume = base area  $\times$  height” works for any prism, and applies to cylinders as well. (Cylinders can be thought of as prisms with circular bases.) The area of a circle is  $\pi r^2$ , so the volume of a cylinder is  $V = \pi r^2 h$ .

The volume formula also works if the cylinder is skewed, as in figure A (right). To see why the volume doesn't change during the skewing process, imagine that the cylinder were composed of a stack of disks (B). If you slide each disk a bit to the right (C), neither the total height nor the total volume of the disks will change.



Many regular solids have volumes that are based on the volume of the enclosing prism. For example, the volume of a pyramid or cone is equal to one-third of the volume of a prism with the same base and height. A sphere has a volume that is two-thirds that of the enclosing cylinder.

For irregular objects, calculating volume cannot be done mathematically. In the 3<sup>rd</sup> century B.C., the Greek mathematician Archimedes devised a clever way to find the volume of *any* object. If you submerge the object in a full tub of water, some of the water will spill out of the tub. If you collect this *displaced* water and measure its volume, you know the volume of the submerged object! You can practice Archimedes' method with your students to find the volume of rocks, shoes, or any object you don't mind getting wet. (Just be sure the object sinks in water.)



## Selected Web Resources

Prism basics: <http://www.mathsisfun.com/geometry/prisms.html>

Archimedes: <http://www.math.nyu.edu/~crrres/Archimedes/contents.html>

Center of mass: [http://en.wikipedia.org/wiki/Center\\_of\\_mass](http://en.wikipedia.org/wiki/Center_of_mass)

*Fido's Flowerbed (Perimeter and Area)* Gizmo: <http://www.explorelearning.com/gizmo/id?1011>

*3D and Orthographic Views* Gizmo: <http://www.explorelearning.com/gizmo/id?281>

*Prisms and Cylinders* Gizmo: <http://www.explorelearning.com/gizmo/id?303>

*Center of Mass* Gizmo: <http://www.explorelearning.com/gizmo/id?606>