Teacher Guide: Adding Decimals

Learning Objectives

Students will…

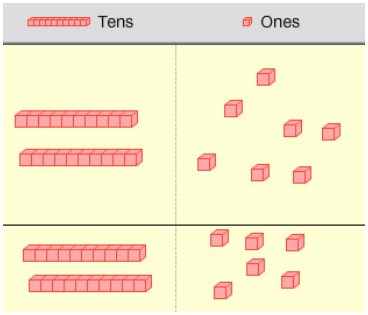
- Add whole numbers with regrouping.
- Add decimals with regrouping.
  - Add decimals with the same number of digits.
  - Add decimals with different numbers of digits.
- Explain what is happening when you "carry" a digit from one column to another during column addition.

Vocabulary

addend, base-10 blocks, base-10 system, decimal, decimal point, sum, whole number

Lesson Overview

The *Adding Decimals* Gizmo™ was designed as a follow-up to the *Modeling Decimals (Base-10 Blocks)* Gizmo. In the *Adding Decimals* Gizmo, students use base-10 blocks to model two addends. They can then model the sum of the two addends. Sets of 10 smaller blocks can be converted to larger blocks to illustrate regrouping.



Modeling 27 + 26

The Student Exploration sheet contains three activities:

- Activity A – Students add whole numbers.
- Activity B – Students add decimals with the same number of digits.
- Activity C – Students add decimals with different numbers of digits.

Suggested Lesson Sequence

1. **Pre-Gizmo activity** (30 – 60 minutes)
   Use the *Modeling Decimals (Base-10 Blocks)* Gizmo to introduce students to modeling numbers with base-10 blocks. If you have access to a set of base-10 blocks for your classroom, you can use these as well. Have students practice modeling whole numbers and decimals using the blocks or the Gizmo.

   The *Adding Decimals* Gizmo was designed to show the logic behind "carrying" in column addition. Review column addition, and ask students what is happening when they carry a digit from one column to another.

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 178 \\ +284 \\ \hline 462 \end{array}$$

2. **Prior to using the Gizmo** (10 – 15 minutes)
   Before students are at the computers, pass out the Student Exploration sheets and ask students to complete the Prior Knowledge Questions. Discuss student answers as a class. At this point, letting students share how they thought about the questions is more valuable than "going over" the correct answers. After the discussion, if possible, use a projector to introduce the Gizmo and demonstrate its basic operations.

3. **Gizmo activity** (15 – 20 minutes per activity)
Assign students to computers. Students can work individually or in small groups. Have students work part of the Student Exploration sheet using the Gizmo. Alternatively, you can use a projector and do the Exploration as a teacher-led activity.

It may be overwhelming for students to do all of the activities in the Student Exploration in one sitting. We recommend starting with the first page of the Student Exploration sheet (Prior Knowledge Questions and Gizmo Warm-up) plus one of the two activities. Extend the lesson if you want using the extensions below. Return to the Gizmo and the unused activities in future class periods to reinforce the concepts.

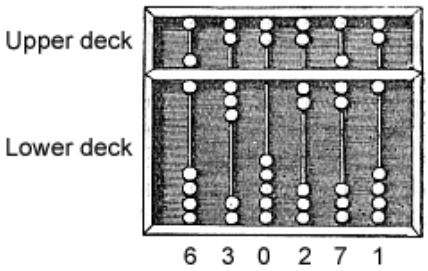
4. **Extending the Gizmo** (15 – 20 minutes each)
Here are some suggestions for extending the activities in the Student Exploration sheet.

Activity A Extension – Project an addition problem such as 45 + 67 on the *Adding Decimals* Gizmo. At the same time, solve the problem on the board using column addition. Show that converting 10 cubes to a rod is equivalent to carrying a "1" from the ones column to the tens column. Then show that converting 10 rods to a flat is equivalent to carrying a "1" from the tens column to the hundreds column.

Activity B Extension – Use the Gizmo to compare adding whole numbers and decimals. For example, add 45 + 67 on the Gizmo, and then add 0.45 and 0.67. Ask students how adding decimals is similar to and different from adding whole numbers.

Activity C Extension – Practice additional problems with so-called "ragged decimals," decimals with different numbers of digits. Ask your students: What is the rule for adding ragged decimals? Students should realize that the key is to line up the decimal points. Once this rule has been established, ask students why it is important that the decimal point is lined up. (It is important to add tenths to tenths, hundredths to hundredths, etc.)

5. **Follow-up activity: Abacus addition** (1 – 3 hours)
A fun way to reinforce concepts of place value is to use an *abacus*, an ancient counting tool. The abacus was invented 5,000 years ago and there are many versions. The Chinese abacus is called the Suanpan.



The Suanpan is divided into two decks. Beads on the upper deck each have a value of 5. Beads on the lower deck each have a value of 1.

Each wire on the Suanpan represents a power of 10. The right wire represents ones, the next column tens, then hundreds, and so forth. (Wires can also represent decimals.) Numbers are modeled by moving beads toward the divider that separates the two decks. In the illustration, the left wire has one upper deck bead (5) and one lower deck bead (1) for a total of 6. The next wire has 3 lower deck beads, so the total is 3. The next wire has no beads near the divider, so the number is 0. The whole number shown is 630,271.

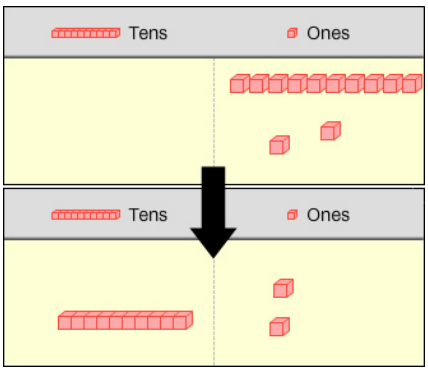
In the classroom, each student or group can create their own abacus from beads, pebbles, string, construction paper, or other materials. (Use your imagination!) You can also use a computer-based abacus simulation. (See **Selected Web Resources**.)

Practice modeling numbers with the abacus, then try simple addition problems. For example, to model 5 plus 7, first model 5 by sliding a single upper deck bead next to the divider. Then add 7 by sliding a second upper deck bead and adding two lower deck beads for a total of 7. The two upper deck beads on the right wire can then be exchanged for a single lower deck bead on the next wire to the left, showing a total of 12. With practice, students will be able to solve multiple-digit addition problems without writing anything down!

Mathematical Background

In the elementary grades, students learn how to add numbers by hand in columns. When the digits in a column add up to more than 10, the "1" is carried to the top of the next column, and the process continues. As students master this procedure, it is unlikely that they will understand what is actually happening when they carry a digit from one column to the next.

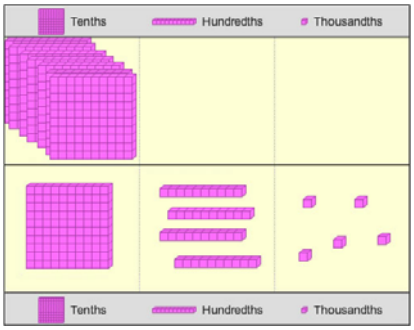
The *Adding Decimals* Gizmo illustrates what carrying a digit means. Using the **100, 10, 1** setting, model two numbers and click **Model sum**. If there are more than ten cubes, ten of the cubes can be converted to a rod in the next column (right). This is equivalent to carrying a 1 from the units column to the tens column. Each rod contains 10 cubes, just as each digit in the tens place represents 10 units in the ones place.



Similarly, ten rods can be changed into a flat. Converting 10 rods to a flat is equivalent to carrying a "1" from the tens place to the hundreds place. A flat contains 10 rods or 100 cubes.

$$\begin{array}{r} 0.7 \\ +\,0.145 \\ \hline 0.845 \end{array}$$

When adding decimals by hand, none of the rules change. The only tricky aspect of adding decimals occurs when the decimals have different numbers of digits. In this case, the rule is to line up the decimal points (left).



The reason for this rule can be illustrated by modeling the two addends on the *Adding Decimals* Gizmo using the **0.1, 0.01, 0.001** setting. After 0.7 and 0.145 are modeled on the Gizmo, it is clear that flats should be added to flats, rods should be added to rods, and cubes should be added to cubes. It would make no sense to add the cubes in the thousandths column to the flats in the tenths column above. When the two sets of blocks are combined, you have eight flats (eight tenths, or 0.8), four rods (four hundredths, of 0.04), and five cubes (five thousandths, or 0.005). The total is 0.845.

Selected Web Resources

Base-10 blocks simulation: http://www.learningbox.com/base10/BaseTen.html
Review of column addition: http://www.mathsisfun.com/numbers/addition-column.html
Addition lesson plan: http://www.athens.edu/vinsobm/lesson_18.htm
Adding with regrouping: http://www.dositey.com/addsub/as85/add3r.htm
Chinese abacus: http://en.wikipedia.org/wiki/Chinese_abacus
Abacus simulation: http://www.mandarintools.com/abacus.html